

North Maharashtra University, Jalgaon

Question Bank

(New syllabus w.e.f. June 2007)

Class: F. Y. B. Sc.

Subject: Mathematics

Paper I

(ALGEBRA AND TRIGNOMETRY)

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Unit – 01

Adjoint and Inverse of Matrix, Rank of a Matrix and Eigen Values and Eigen Vectors

Marks – 02

- 1) If $A = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 2 & 1 \\ 3 & 1 & 0 \end{bmatrix}$, find minor and cofactor of a_{11} , a_{23} and a_{32}
- 2) If $A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$, find adj A
- 3) If $A = \begin{bmatrix} -1 & 3 \\ 7 & 2 \end{bmatrix}$, find A^{-1}
- 4) If $A = \begin{bmatrix} -1 & 2 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix}$, find $\rho(AB)$
- 5) If $A = \begin{bmatrix} 1 & 0 & 1 \\ -2 & 5 & 7 \\ 1 & 2 & 3 \end{bmatrix}$, find $\rho(A)$
- 6) Find rank of $A = \begin{bmatrix} 2 & 6 \\ 1 & 3 \\ 3 & 9 \end{bmatrix}$
- 7) Find the characteristic equation and eigen values of $A = \begin{bmatrix} 9 & -7 \\ 3 & -1 \end{bmatrix}$
- 8) Define characteristic equation of a matrix A and state Cayley-Hamilton Theorem.
- 9) Define adjoint of a matrix A and give the formula for A^{-1} if it exist.
- 10) Define inverse of a matrix and state the necessary and sufficient condition for existence of a matrix.
- 11) Compute $E_{12}(3)$, $E_2'(3)$ of order 3

12) If $A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 6 \\ -3 & 2 \end{bmatrix}$, find $(AB)^{-1}$

13) If $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ then $\rho(A)$ is ---

- a) 0 b) 1 c) 2 d) 4

14) If $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ then which of the following is true ?

- a) $\text{adj}A$ is nonsingular b) $\text{adj}A$ has a zero row
 c) $\text{adj}A$ is symmetric d) $\text{adj}A$ is not symmetric

15) If $A = \begin{bmatrix} -1 & -2 \\ -3 & 4 \end{bmatrix}$ then which of the following is true ?

- a) $A^2 = A$ b) A^2 is identity matrix
 c) A^2 is non-singular d) A^2 is singular

16) If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$

Statement I : AB singular

Statement II : $\text{adj}(AB) = \text{adj}B \text{adj}A$

then which of the following is true

- a) Statement I is true b) Statement II is true
 c) Both Statements are true d) both statements are false

17) If A is a square matrix, then A^{-1} exists iff

- a) $|A| > 0$ b) $|A| < 0$
 c) $|A| = 0$ d) $|A| \neq 0$

18) If $A = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ then $A(\text{adj} A)$ is

- a) $\begin{bmatrix} 0 & 10 \\ 10 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix}$
 c) $\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$ d) $\begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$

- 27) If A is symmetric matrix then
- a) $\text{adj}A$ is non-singular matrix b) $\text{adj}A$ is symmetric matrix
c) $\text{adj}A$ does not exist d) None of these
- 28) If $AB = \begin{bmatrix} 1 & 3 \\ -2 & 1 \end{bmatrix}$ and $A = \begin{bmatrix} 3 & -7 \\ 4 & -2 \end{bmatrix}$ then
- a) $(AB)^{-1} = AB$ b) $(AB)^{-1} = A^{-1}B^{-1}$
c) $(AB)^{-1} = B^{-1}A^{-1}$ d) None of these
- 29) If $|A| \neq 0$ and B, C are matrices such that $AB = AC$ then
- a) $B \neq C$ b) $B \neq A$ c) $B = C$ d) $C \neq A$
- 30) If $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ then
- a) $A^2 = I$ b) $A^2 = 0$ c) $A^2 = A$ d) None of these
- 31) If matrix A is equivalent to matrix B then
- a) $\rho(A) \neq \rho(B)$ b) $\rho(A) > \rho(B)$
c) $\rho(A) = \rho(B)$ d) None of these
- 32) If $A = [1 \ -4 \ 0]$ then $\rho(A)$ is
- a) 0 b) 1 c) 3 d) None of these
- 33) If $A = \begin{bmatrix} 1 & 9 & 2 & 0 \\ 0 & -3 & 4 & 1 \\ 1 & 9 & 2 & 0 \end{bmatrix}$ then $\rho(A)$ is
- a) 0 b) 1 c) 2 d) 3
- 34) If A is a matrix of order $m \times n$ then
- a) $\rho(A) \leq \min\{m,n\}$ b) $\rho(A) \leq \min\{m,n\}$
c) $\rho(A) \geq \max\{m,n\}$ d) None of these
- 35) The eigen values of $A = \begin{bmatrix} -2 & 7 \\ 2 & 3 \end{bmatrix}$ are
- a) -5, -4 b) 5, 4 c) 5, -4 d) None of these

36) If $A = \begin{bmatrix} 1 & -5 \\ 3 & 2 \end{bmatrix}$ then A satisfies

- a) $A^2 + 3A + 17I = 0$ b) $A^2 - 3A - 17I = 0$
c) $A^2 - 3A + 17I = 0$ d) $A^2 + 3A - 17I = 0$

37) If A is a matrix and λ is some scalar such that $A - \lambda I$ is singular then

- a) λ is eigen value of A b) λ is not an eigen value of A
c) $\lambda = 0$ d) None of these

38) If $A = \begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ then A^{-1} exists if

- a) $\rho(A) = 0$ b) $\rho(A) = 3$ c) $\rho(A) = 1$ d) None of these

39) If $A = \begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix}$ then which of the following is incorrect ?

- a) $A = A^{-1}$ b) $A^2 = I$ c) $A^2 = 0$ d) None of these

Marks : 04

- 1) If A is a square matrix of order n then prove that $(\text{adj}A)' = \text{adj} A'$

and verify it for $A = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$

- 2) For the following matrix, verify that $(\text{adj}A)' = \text{adj} A'$

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

- 3) If $A = \begin{bmatrix} 4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$ then show that $\text{adj} A = A$

- 4) If $A = \begin{bmatrix} -3 & 1 & 0 \\ 2 & -2 & 1 \\ -1 & -1 & 1 \end{bmatrix}$, show that $A(\text{adj} A)$ is null matrix.

- 5) Show that the adjoint of a symmetric matrix is symmetric and verify it for

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

- 6) Verify that $(\text{adj}A)A = |A|I$ for the matrix $A = \begin{bmatrix} 4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$

- 7) Verify that $A(\text{adj}A) = (\text{adj}A)A = |A|I$ for the matrix $A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$

- 8) Verify that $A(\text{adj}A) = |A|I$ for the matrix $A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{bmatrix}$

- 9) Find the inverse of $A = \begin{bmatrix} -1 & -2 & -1 \\ 2 & 1 & 0 \\ -3 & 1 & -1 \end{bmatrix}$

10) Find the inverse of $A = \begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ -3 & -1 & 1 \end{bmatrix}$

11) Show that the matrix $A = \begin{bmatrix} 4 & -1 \\ -3 & 2 \end{bmatrix}$ satisfies the equation $A^2 - 6A + 5I = 0$.

Hence find A^{-1}

12) If $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 3 \\ 7 & 2 \end{bmatrix}$, show that $\text{adj}(AB) = \text{adj}B \text{adj}A$

13) If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, show that $A(\text{adj}A) = (\text{adj}A)A = |A|I$

14) If $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ then show that $\text{adj}A = 3A'$

15) If $A = \begin{bmatrix} -2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$, show that $A^2 = A$, but A^{-1} does not exist.

16) If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$, show that $A^3 = A^{-1}$

17) What is the reciprocal of the following matrix ?

$$A = \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

18) If $A = \begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1} A^{-1}$

19) Using adjoint method find the inverse of the matrix $A = \begin{bmatrix} -1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$

20) If A is a non-singular matrix of order n then prove that $\text{adj}(\text{adj}A) = |A|^{n-2} A$

21) For a non-singular square matrix A of order n , prove that

$$|\text{adj}(\text{adj} A)| = |A|^{(n-1)^2}$$

22) For a non-singular square matrix A of order n , prove that

$$\text{adj} \{ \text{adj}(\text{adj} A) \} = |A|^{n^2 - 3n + 3} A^{-1}$$

23) If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, show that $A^3 = A^{-1}$

24) Find the rank of the matrix $A = \begin{bmatrix} 2 & 3 & 2 \\ 3 & 2 & 3 \\ 1 & 4 & 1 \end{bmatrix}$

25) Find the rank of the matrix $A = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 3 & 4 \\ 3 & 2 & 4 \end{bmatrix}$

26) Compute the elementary matrix $[E_2(-3)]^{-1} \cdot E_{31}(2) \cdot E'_{21}(1/2)$ of order 3

27) Compute the matrix $E'_{2}(1/3) \cdot E_{31} \cdot [E_2(-4)]^{-1}$ for E-matrices of order 3

28) Determine the values of x so that the matrix $\begin{bmatrix} x & x & 2 \\ 2 & x & x \\ x & 2 & x \end{bmatrix}$ is of

i) rank 3 ii) rank 2 iii) rank 1

29) Determine the values of x so that the matrix $\begin{bmatrix} x & x & 1 \\ 1 & x & x \\ x & 1 & x \end{bmatrix}$ is of

i) rank 3 ii) rank 2 iii) rank 1

30) Reduce the matrix A to the normal form. Hence determine its rank,

where $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 6 & 8 & 10 \end{bmatrix}$

- 31) Reduce the matrix A to the normal form. Hence determine its rank,

$$\text{where } A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 1 & 2 & 3 & 4 \\ 2 & 6 & 7 & 5 \end{bmatrix}$$

- 32) Reduce the matrix A to the normal form. Hence determine its rank,

$$\text{where } A = \begin{bmatrix} 3 & 2 & 5 & 7 & 12 \\ 1 & 1 & 2 & 3 & 5 \\ 3 & 3 & 6 & 9 & 15 \end{bmatrix}$$

- 33) Find non-singular matrices P and Q such that PAQ is in normal form,

$$\text{where } A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & -1 & -1 \end{bmatrix}$$

- 34) Find non-singular matrices P and Q such that PAQ is in normal form,

$$\text{where } A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$$

- 35) Find non-singular matrices P and Q such that PAQ is in normal form,

$$\text{where } A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix} \text{ Also find } \rho(A)$$

- 36) Show that the matrix $A = \begin{bmatrix} x-1 & 1 & 2 \\ 0 & x & 4 \\ -3 & 2 & x \end{bmatrix}$

has rank 3 when $x \neq 2$ and $x \neq \pm\sqrt{2}$, find its rank when $x = 2$.

- 37) Find a non-singular matrix P such that $PA = \begin{bmatrix} G \\ 0 \end{bmatrix}$ for the matrix

$$A = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 3 & 4 \\ 3 & 2 & 4 \end{bmatrix} \text{ Hence find } \rho(A).$$

38) Given $A = \begin{bmatrix} -1 & -2 & -1 \\ 6 & 12 & 6 \\ 5 & 10 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$,

verify that $\rho(AB) \leq \min \{\rho(A), \rho(B)\}$

39) Find all values of θ in $[-\pi/2, \pi/2]$ such that the matrix

$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & \sin \theta \\ 0 & \sin \theta & \cos \theta \end{bmatrix}$ is of rank 2.

40) Express the following non-singular matrix A as a product of E – matrices,

where $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$

41) Express the following non-singular matrix A as a product of E – matrices,

where $A = \begin{bmatrix} 7 & 0 & 3 \\ 0 & 1 & 0 \\ 2 & 0 & -1 \end{bmatrix}$

42) Express the following non-singular matrix A as a product of E – matrices,

where $A = \begin{bmatrix} 13 & 3 & 3 \\ 4 & 1 & 1 \\ 4 & 0 & 1 \end{bmatrix}$

43) State Cayley- Hamilton Theorem. Verify it for $A = \begin{bmatrix} 1 & -5 \\ 3 & 2 \end{bmatrix}$

44) State Cayley- Hamilton Theorem. Verify it for $A = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$

45) Verify Cayley- Hamilton Theorem for $A = \begin{bmatrix} 1 & 2 & 0 \\ -3 & -2 & 1 \\ 1 & 3 & -1 \end{bmatrix}$

46) Find the characteristics equation of $A = \begin{bmatrix} 3 & 2 & -1 \\ 1 & 3 & 0 \\ 2 & -1 & 2 \end{bmatrix}$

- 47) Find eigen values of $A = \begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}$
- 48) If λ is a non-zero eigen value of a non-singular matrix A , show that $1/\lambda$ is an eigen value of A^{-1}
- 49) If $\lambda \neq 0$ is an eigen value of a non-singular matrix A , show that $|A|/\lambda$ is an eigen value of $\text{adj } A$.
- 50) Let k be a non-zero scalar and A be a non-zero square matrix, show that if λ is an eigen value of A then λk is an eigen value of kA .
- 51) Let A be a square matrix. Show that 0 is an eigen value of A iff A is singular.

52) Show that $A = \begin{bmatrix} 0 & a & b \\ a & 0 & c \\ b & c & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & b & a \\ b & 0 & c \\ a & c & 0 \end{bmatrix}$

have the same characteristic equation.

- 53) Find eigen values and corresponding eigen vectors of $A = \begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$
- 54) Find eigen values and corresponding eigen vectors of $A = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}$
- 55) Find characteristic equation of $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$

Also find A^{-1} by using Cayley Hamilton theorem.

- 56) Verify Cayley Hamilton theorem for A and hence find A^{-1}

where $A = \begin{bmatrix} -2 & 7 \\ 3 & 4 \end{bmatrix}$

Marks - 04 / 06

- 1) If A, B are matrices such that product AB is defined then prove that $(AB)' = B' A'$
- 2) If $A = [a_{ij}]$ is a square matrix of order n then show that $A(\text{adj}A) = (\text{adj}A)A = |A|I$
- 3) Show that a square matrix A is invertible if and only if $|A| \neq 0$
- 4) If A, B are non-singular matrices of order n then prove that AB is non-singular and $(AB)^{-1} = B^{-1} A^{-1}$
- 5) If A, B are non-singular matrices of same order then prove that $\text{adj}(AB) = (\text{adj}B) (\text{adj}A)$
- 6) If A is a non-singular matrix then prove that $(A^n)^{-1} = (A^{-1})^n, \forall n \in \mathbb{N}$
- 7) If A is a non-singular matrix and $k \neq 0$ then prove that $(kA)^{-1} = \frac{1}{k} A^{-1}$
- 8) If A is a non-singular matrix then prove that $(\text{adj} A)^{-1} = \text{adj} A^{-1} = \frac{A}{|A|}$
- 9) State and prove the necessary and sufficient condition for a square matrix A to have an inverse.
- 10) If A is a non-singular matrix then show that $AB = AC$ implies $B = C$
Is the result true when A is singular? Justify.
- 11) When does the inverse of a matrix exist? Prove that the inverse of a matrix, if it exists, is unique.
- 12) If a non-singular matrix A is symmetric prove that A^{-1} is also symmetric.
- 13) Prove that inverse of an elementary matrix is an elementary matrix of the same type.
- 14) If A is a $m \times n$ matrix of rank r , prove that there exist non-singular matrices P and Q such that $PAQ = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$
- 15) Prove that every non-singular matrix can be expressed as a product of finite number of elementary matrices.

- 16) If A is an $m \times n$ matrix of rank r , then show that there exists a non-singular matrix P such that $PA = \begin{bmatrix} G \\ 0 \end{bmatrix}$, where G is $r \times n$ matrix of rank r and 0 is null matrix of order $(m-r) \times n$.
- 17) Prove that the rank of the product of two matrices can not exceed the rank of either matrix.
- 18) If A is an $m \times n$ matrix of rank r then show that there exists a non-singular matrix Q such that $AQ = \begin{bmatrix} H & 0 \end{bmatrix}$ Where H is $m \times r$ matrix of rank r and 0 is null matrix of order $m \times (n-r)$.

Unit - 02

System of Linear Equations and Theory of Equations

Marks - 02

- 1) Examine for non-trivial solutions
 $x + y + z = 0$
 $4x + y = 0$
 $2x + 2y + 3z = 0$
- 2) Define i) Consistent and inconsistent system ii) Equivalent system
- 3) Define homogeneous, non-homogeneous system of equations.
- 4) The equation $x^4 + 4x^3 - 2x^2 - 12x + 9 = 0$ has two pairs of equal roots, find them.
- 5) Change the signs of the roots of the equation $x^7 + 5x^5 - x^3 + x^2 + 7x + 3 = 0$
- 6) Transform the equation $x^7 - 7x^6 - 3x^4 + 4x^2 - 3x - 2 = 0$ into another whose roots shall be equal in magnitude but opposite in sign to those of this equation.
- 7) Change of the equation $3x^4 - 4x^3 + 4x^2 - 2x + 1 = 0$ into another the coefficient of whose highest term will be unity.
- 8) A system $AX = B$, of m linear equations in n unknowns, is consistent iff
 - A) $\text{rank}A \neq \text{rank} [A, B]$
 - B) $\text{rank}A = \text{rank} [A, B]$
 - C) $\text{rank}A \geq \text{rank} [A, B]$
 - D) $\text{rank}A \leq \text{rank} [A, B]$
- 9) For the equation $x^4 + x^2 + x + 1 = 0$, sum of roots taken one, two, three and four at a time is respectively.
 - A) 1, 1, 1, 1
 - B) 0, 1, -1, 1
 - C) 1, 0, -1, 1
 - D) -1, 1, -1, 1
- 10) For the equation $x^4 + x^3 + x^2 + x + 1 = 0$, sum of roots taken one, two, three and four at a time is respectively.
 - A) 1, 1, 1, 1
 - B) -1, 1, -1, 1
 - C) 1, -1, 1, -1
 - D) -1, -1, -1, 1

Marks - 04

1. Examine for consistency the following system of equations

$$x + z = 2$$

$$-2x + y + 3z = 3$$

$$-3x + 2y + 7z = 4$$

2. Solve the following system of equations

$$x + y + z = 6$$

$$2x + y + 3z = 13$$

$$5x + 2y + z = 12$$

$$2x - 3y - 2z = -10$$

3. If $A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & -2 & 3 \\ -1 & 3 & -4 \end{bmatrix}$, find A^{-1} . Hence solve the following system of linear

equations $2x + y - z = 1$ $x - 2y + 3z = 9$ $-x + 3y - 4z = -12$

4. Test the following equations for consistency and if consistent solve them

$$2x - y - 5z + 4w = 1$$

$$x + 3y + z - 5w = 18$$

$$3x - 2y - 8z + 7w = -1$$

5. Solve the following system of equations

$$x_1 + 3x_2 + 4x_3 - 6x_4 = 0$$

$$x_2 + 6x_3 = 0$$

$$2x_1 + 2x_2 + 2x_3 - 3x_4 = 0$$

$$x_1 + x_2 - 4x_3 - 4x_4 = 0$$

6. Examine for non-trivial solutions the following homogeneous system of linear equations

$$x + y + 3z = 0$$

$$x - y + z = 0$$

$$-x + 2y = 0$$

$$x - y + z = 0$$

7. Solve the system of equations

$$x + 3y + 3z = 14$$

$$x + 4y + 3z = 16$$

$$x + 3y + 4z = 17$$

by i) method of inversion ii) method of reduction.

8. Examine the following systems of equation for consistency

$$x - 2y + z - u = 1$$

$$x + y - 2z + 3u = -2$$

$$4x + y - 5z + 8u = -5$$

$$5x - 7y + 2z - u = 3$$

9. Test the following equations for consistency and solve them

$$x + 2y + z = 2$$

$$3x + y - 2z = 1$$

$$4x - 3y - z = 3$$

$$x + 2y + z = 2$$

10. Solve the following equations

$$4u + 2v + w + 3t = 0$$

$$2u + v + t = 0$$

$$6u + 3v + 4w + 7t = 0$$

11. Solve the equation $x^3 - 3x^2 - 6x + 8 = 0$ if the roots are in A.P.

12. Solve the equation $x^3 - 9x^2 + 14x + 24 = 0$ if two of its roots are in the ratio 3:2.

13. Solve the equation $3x^3 - 26x^2 + 52x - 24 = 0$ if the roots are in G.P.

14. Solve the equation $x^4 + 2x^3 - 21x^2 - 22x + 40 = 0$ whose roots are in A.P.

15. If α , β and γ are roots of the equation $x^3 - 5x^2 - 2x + 24 = 0$ find the value of

i) $\sum \alpha^2\beta$

ii) $\sum \alpha^2$

iii) $\sum \alpha^3$

iv) $\sum \alpha^2\beta^2$

16. Remove the fractional coefficients from the equation $x^3 - \frac{1}{2}x^2 + \frac{2}{3}x - 1 = 0$

17. Remove the fractional coefficients from the equation $x^3 - \frac{5}{2}x^2 - \frac{7}{18}x + \frac{1}{108} = 0$
18. Transform the equation $5x^3 - \frac{3}{2}x^2 - \frac{3}{4}x + 1 = 0$ to another with integral coefficients and unity for the coefficient of the first term.
19. Remove the fractional coefficients from the equation

$$x^4 + \frac{3}{10}x^2 + \frac{13}{25}x + \frac{77}{1000} = 0$$
20. Find the equation whose roots are reciprocals of the roots
of $x^4 - 5x^3 + 7x^2 + 3x - 7 = 0$
21. Find the equation whose roots are the roots of $x^4 - 5x^3 + 7x^2 - 17x + 11 = 0$
each diminished by 4.
22. Find the equation whose roots are those of $3x^3 - 2x^2 + x - 9 = 0$ each
diminished by 5.
23. Remove the second term from equation $x^4 - 8x^3 + x^2 - x + 3 = 0$
24. Remove the third term of equation $x^4 - 4x^3 - 18x^2 - 3x + 2 = 0$, hence obtain
the transformed equation in case $h = 3$.
25. Transform the equation $x^4 + 8x^3 + x - 5 = 0$ into one in which the second term
is vanishing.
26. Solve the equation $x^4 + 16x^3 + 83x^2 + 152x + 84 = 0$ by removing the second term.
27. Solve the equation $x^3 + 6x^2 + 9x + 4 = 0$ by Carden's method.
28. Solve the equation $x^3 - 15x^2 - 33x + 847 = 0$ by Carden's method.
29. Solve the equation $z^3 - 6z^2 - 9 = 0$ by Carden's method.
30. Solve the equation $x^3 - 21x - 344 = 0$ by Carden's method.
31. Solve $x^3 - 15x - 126 = 0$ by Carden's method
32. Solve $27x^3 - 54x^2 + 198x - 73 = 0$ by Carden's method
33. Solve $x^3 + 3x^2 - 27x + 104 = 0$ by Carden's method
34. Solve $x^3 - 3x^2 + 12x + 16 = 0$ by Carden's method
35. Solve $x^4 - 5x^2 - 6x - 5 = 0$ by Descarte's method.
36. Solve the biquadratic $x^4 + 12x - 5 = 0$ by Descarte's method.
37. Solve $x^4 - 8x^2 - 24x + 7 = 0$ by Descarte's method.

Marks - 04 / 06

- For what values of a , the equations
$$x + y + z = 1$$
$$2x + 3y + z = a$$
$$4x + 9y - z = a^2$$
 have a solution and solve then completely in each case.
- Investigate for what values of λ and μ the following system of equations
$$x + 3y + 2z = 2$$
$$2x + 7y - 3z = -11$$
$$x + y + \lambda z = \mu$$
 have
 - No solution
 - A unique solution
 - Infinite number of solutions.
- Show that the system of equations
$$ax + by + cz = 0$$
$$bx + cy + az = 0$$
$$cx + ay + bz = 0$$
 has a non-trivial solution iff $a + b + c = 0$ or $a = b = c$
- Find the value of λ for which the following system have a non-trivial solution
$$x + 2y + 3z = 0$$
$$2x + 3y + 4z = 0$$
$$3x + 4y + \lambda z = 0$$
- Discuss the solutions of system of equations
$$(5 - \lambda)x + 4y = 0$$
$$x + (2 - \lambda)y = 0$$
 for all values of λ .
- Obtain the relation between the roots and coefficients of general polynomial equation $a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \dots + a_{n-1}x + a_n = 0$
- Solve the equation $x^3 - 5x^2 - 16x + 80 = 0$ if the sum of two of its roots being equal to zero.
- Solve the equation $x^3 - 3x^2 + 4 = 0$ if the two of its roots are equal.
- Solve the equation $x^3 - 5x^2 - 2x + 24 = 0$ if the product of two of the roots is 12.

10. Solve the equation $x^3 - 7x^2 + 36 = 0$ if one root is double of another.
11. Find the condition that the roots of the equation $x^3 - px^2 + qx - r = 0$ are in A.P.
12. Find the condition that the cubic equation $x^3 + px^2 + qx + r = 0$ should have two roots α and β connected by the relation $\alpha\beta + 1 = 0$
13. If α, β and γ are roots of the cubic equation $x^3 + px^2 + qx + r = 0$ find the value of i) $\sum \alpha^2\beta$ ii) $\sum \alpha^2$ iii) $\sum \alpha^3$ iv) $\sum \alpha^2\beta^2$
14. If α, β and γ are roots of the cubic equation $x^3 + px^2 + qx + r = 0$ find the value of $(\beta+\gamma)(\gamma+\alpha)(\alpha+\beta)$
15. If α, β and γ are roots of the cubic equation $x^3 - px^2 + qx - r = 0$ find the value of $\frac{1}{\beta^2\gamma^2} + \frac{1}{\gamma^2\alpha^2} + \frac{1}{\alpha^2\beta^2}$
16. If α, β, γ and δ are roots of biquadratic equation $x^4 + px^3 + qx^2 + rx + s = 0$, find the value of the following symmetric functions
i) $\sum \alpha^2\beta$ ii) $\sum \alpha^2$ iii) $\sum \alpha^3$
17. If α, β, γ and δ are roots of biquadratic equation $x^4 + px^3 + qx^2 + rx + s = 0$, find the value of the following symmetric functions
i) $\sum \alpha^2\beta\gamma$ ii) $\sum \alpha^2\beta^2$ iii) $\sum \alpha^4$
18. Remove the fractional coefficients from the equation
$$x^4 - \frac{5}{6}x^3 + \frac{5}{12}x^2 - \frac{13}{900} = 0$$
19. Find the equation whose roots are the reciprocals of the roots of
$$x^4 - 3x^3 + 7x^2 + 5x - 2 = 0$$
20. Transform an equation $a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \dots + a_{n-1}x + a_n = 0$ into another whose roots are the roots of given equation diminished by given quantity h .
21. If α, β, γ are the roots of $8x^3 - 4x^2 + 6x - 1 = 0$ find the equation whose roots are $\alpha + 1/2, \beta + 1/2, \gamma + 1/2$
22. Solve the equation $x^4 + 20x^3 + 143x^2 + 430x + 462 = 0$ by removing its second term.
23. Reduce the cubic $2x^3 - 3x^2 + 6x - 1 = 0$ to the form $Z^3 + 3HZ + G = 0$
24. Explain Carden's method of solving equation $a_0x^3 + 3a_1x^2 + 3a_2x + a_3 = 0$

Unit – 03

Relations, Congruence Classes and Groups

Marks - 02

- 1) Let $A = \{ 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 \}$
 $A_1 = \{ 1, 2, 3, 4 \}$, $A_2 = \{ 5, 6, 7 \}$, $A_3 = \{ 4, 5, 7, 9 \}$, $A_4 = \{ 4, 8, 10 \}$,
 $A_5 = \{ 8, 9, 10 \}$, $A_6 = \{ 1, 2, 3, 6, 8, 10 \}$ Which of the following is the partition of A.
- A) $\{ A_1, A_2, A_5 \}$ B) $\{ A_1, A_3, A_5 \}$
C) $\{ A_2, A_3, A_6 \}$ D) $\{ A_2, A_3, A_6 \}$
- 2) Let $A = \mathbb{Z}^+$, the set of all positive integers. Define a relation on A as “aRb iff a divides b” then this relation is not ---
- A) Reflexive B) Symmetric
C) Transitive D) Antisymmetric
- 3) For $n \in \mathbb{N}$, $a, b \in \mathbb{Z}$ and $d = (a, n)$, linear congruence $ax \equiv b \pmod{n}$ has a solution iff ----
- A) $d \mid b$ B) $x \mid b$
C) $n \mid d$ D) $a \mid b$
- 4) If the Linear congruence $ax \equiv b \pmod{n}$ has a solution then it has exactly ---- non-congruent modulo n solutions
- A) a B) b
C) n D) (a, n)
- 5) If $a^2 \equiv b^2 \pmod{p}$ then $p \mid a+b$ or $p \mid a-b$ only when p is ---
- A) Even B) Odd
C) Prime D) Composite
- 6) $G = \{ 1, -1 \}$ is a group w.r.t. usual
- A) Addition B) Subtraction
C) Multiplication D) None of these

- 7) In the group $(\mathbb{Z}_6, +_6)$, $o(\bar{5})$ is
 A) 2 B) 5 C) 6 D) 1
- 8) Linear congruence $207x \equiv 6 \pmod{18}$ has
 A) No solution B) Nine solutions
 C) Three solutions D) One solution
- 9) The number of residue classes of integers modulo 7 are
 A) one B) five C) six D) seven
- 10) The solution of the linear congruence $5x \equiv 2 \pmod{7}$ is
 A) $x = 2$ B) $x = 4$ C) $x = 6$ D) $x = 3$
- 11) The set of positive integers under usual multiplication is not a group as following does not exist
 A) identity B) inverse
 C) associativity D) commutativity
- 12) Define an equivalence relation and show that ' $>$ ' on set of naturals is not an equivalence relation.
- 13) Define a partition of a set and find any two partitions of $A = \{a, b, c, d\}$
- 14) Define equivalence class of an element. Find equivalence classe of '2' if $R = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (3, 1), (2, 3), (3, 3), (4, 4), (3, 2), (5, 5)\}$ is an equivalence relation on $A = \{1, 2, 3, 4, 5\}$
- 15) Define residue classes of integers modulo n. Find the residue class of $\bar{2}$ for the relation "congruence modulo 5".
- 16) Define prime residue class modulo n. Find the prime residue class modulo 7
- 17) Define a group and show that set of integers with respect to usual multiplication is not a group.
- 18) Define Abelian group and show that group
 $G = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : ad - bc \neq 0, a, b, c, d \in R \right\}$ is not abelian.
- 19) Define finite and infinite group. Illustrate by an example.

- 20) Define order of an element and find order of each element in a group
 $G = \{1, -1, i, -i\}$ under multiplication.
- 21) Find any four partitions of the set $S = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$
- 22) Show that $A \times B \neq B \times A$ if $A = \{2, 4, 6\}$, $B = \{7, 9, 11\}$
- 23) In the group (\mathbb{Z}'_8, X_8) , find order of $\bar{3}$, $\bar{4}$, $\bar{5}$, $\bar{6}$
- 24) Let $\mathbb{Z}'_8 = \{\bar{1}, \bar{3}, \bar{5}, \bar{7}\}$, find $(\bar{3})^4$, $(\bar{3})^0$, $(\bar{3})^{-4}$ in a group (\mathbb{Z}'_8, X_8)
- 25) In the group (\mathbb{Z}'_7, X_7) , find $(\bar{3})^2$, $(\bar{4})^{-3}$, $o(\bar{3})$, $o(\bar{4})$
- 26) Find domain and range of a relation $R = \{(x, y) : x \mid y \text{ for } x \in A, y \in B\}$
where $A = \{2, 3, 7, 8\}$, $B = \{4, 6, 9, 14\}$
- 27) Solve the linear congruence $2x + 1 \equiv 4 \pmod{5}$
- 28) Let $X = \{1, 2, 3\}$ and $R = \{(1,1), (1, 2), (2, 1), (2, 2), (3, 3), (1, 3), (3, 1), (2, 3), (3, 2)\}$ Is the relation R reflexive, symmetric and transitive ?
- 29) Prepare the multiplication table for the set of prime residue classes modulo 12.
- 30) Show that in a group G every element has unique inverse.
- 31) Show that the linear congruence $13x \equiv 9 \pmod{25}$ has only one solution.
- 32) Show that the linear congruence $4x \equiv 11 \pmod{6}$ has no solution.
- 33) If $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$ then show that $ac \equiv bd \pmod{n}$
- 34) A relation R is defined in the set Z of all integers as “ aRb iff $7a - 4b$ is divisible by 3”. Prove that R is symmetric.
- 35) Let \sim be an equivalence relation on a set A and $a, b \in A$. Show that $b \in [a]$ iff $[a] = [b]$
- 36) If in a group G, every element is its own inverse then prove that G is abelian.
- 37) In a group every element except identity element is of order two. Show that G is abelian.
- 38) If R and S are equivalence relations in set X. Prove that $R \cap S$ is an equivalence relation.
- 39) In the set R of all real numbers, a relation \sim is defined by $a \sim b$ if $2 + ab > 0$. Show that \sim is reflexive, symmetric and not transitive.

Marks - 04

1. Let Z be the set of all integers. Define a relation R on Z by xRy iff $x-y$ is an even integer. Show that R is an equivalence relation.
2. Let P be the set of all people living in a Jalgaon city. Show that the relation “has the same surname as” on P is an equivalence relation.
3. Let $P(X)$ be the collection of all subsets of X (power set of X). Show that the relation “is a proper subset of” in $P(X)$ is not an equivalence relation.
4. Show that in the set of integers $x \sim y$ iff $x^2 = y^2$ is an equivalence relation and find the equivalence classes.
5. Let S be the set of points in the plane. For any two points $x, y \in S$, define $x \sim y$ if distances of x and y is same from origin. Show that \sim is an equivalence relation. What are the equivalence classes?
6. Consider the set $N \times N$. Define $(a, b) \sim (c, d)$ iff $ad = bc$. Show that \sim is an equivalence relation. What are the equivalence classes?
7. Find the composition table for
 - i) $(Z_5, +_5)$ and (Z_5, \times_5)
 - ii) $(Z_7, +_7)$ and (Z_7, \times_7)
8. Prepare the composition tables for addition and multiplication of $Z_6 = \{ \bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{5} \}$
9. Show that $\bar{a} \in Z_n$ has a multiplicative inverse in Z_n iff $(a, n) = 1$
10. Find the remainder when 8^{103} is divided by 13.
11. Show that $G = \{ 1, -1, i, -i \}$, where $i = \sqrt{-1}$, is an abelian group w.r.t. usual multiplication of complex numbers.
12. Show that the set of all 2×2 matrices with real numbers w.r.t multiplication of matrices is not a group.
13. Show that $G = \{ A : A \text{ is non-singular matrix of order } n \text{ over } R \}$ is a group w.r.t. usual multiplication of matrices.
14. Let Q^+ denote the set of all positive rationals. For $a, b \in Q^+$ define $a * b = \frac{ab}{2}$
Show that $(Q^+, *)$ is a group.

15. Show that $G = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : ad - bc \neq 0 \text{ \& } a, b, c, d \in \mathbb{R} \right\}$ w.r.t. matrix multiplication is a group but it is not an abelian group.
16. Let Z_n be the set of residue classes modulo n with a binary operation $a+_nb = \overline{a+b} = \bar{r}$ where \bar{r} is the remainder when $a + b$ is divided by n Show that $(Z_n, +_n)$ is a finite abelian group.
17. Let Z'_n denote the set of all prime residue classes modulo n . Show that Z'_n is an abelian group of order $\phi(n)$ w.r.t. \times_n
18. Let G be a group and $a, b \in G$ be such that $ab = ba$. Prove that $(ab)^n = a^n b^n, n \in \mathbb{Z}$
19. If in a group G every element is its own inverse then prove that G is an abelian group.
20. Let $G = \{ (a, b) / a, b \in \mathbb{R}, a \neq 0 \}$ Define \odot on G as $(a, b) \odot (c, d) = (ac, bc + d)$. Show that (G, \odot) is a non-abelian group.
21. Let f_1, f_2 be real valued functions defined by $f_1(x) = x$ and $f_2(x) = 1-x, \forall x \in \mathbb{R}$. Show that $G = \{ f_1, f_2 \}$ is group w.r.t. composition of mappings.
22. Let G be a group and $\forall a, b \in G, (ab)^n = a^n b^n$ for three consecutive integers n . Show that G is an abelian group.
23. Show that a group G is abelian iff $(ab)^2 = a^2 b^2, \forall a, b \in G$
24. Prove that a group having 4 elements must be abelian.
25. Using Fermat's Theorem, Show that $5^{10} - 3^{10}$ is divisible by 11.
26. Using Fermat's Theorem find the remainder when 2^{105} is divided by 11.
27. Solve
i) $8x \equiv 6 \pmod{14}$ ii) $13x \equiv 9 \pmod{25}$
28. Let $*$ be an operation defined by $a * b = a + b + 1 \forall a, b \in \mathbb{Z}$ where \mathbb{Z} is the set of integers. Show that $\langle \mathbb{Z}, * \rangle$ is an abelian group.
29. Let $A_\alpha = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, where $\alpha \in \mathbb{R}$ and $G = \{ A_\alpha : \alpha \in \mathbb{R} \}$. Prove that G is an abelian group under multiplication of matrices.

30. Let Q^+ be the set of all positive real numbers and define $*$ on Q^+ by $a*b = \frac{ab}{3}$. Show that $(Q^+, *)$ is an abelian group.
31. Find the remainder when $2^{73} + 14^3$ is divided by 11.
32. Show that the set $G = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \right\}$ is a group w.r.t. multiplication.
33. Show that $G = \mathbb{R} - \{1\}$ is an abelian group under the binary operation $a * b = a + b - ab, \forall a, b \in G$
34. If the elements a, b and ab of a finite group G are each of order 2 then show that $ab = ba$.
35. A relation R is defined in the set of integers Z by xRy iff $7x - 3y$ is divisible by 4. Show that R is an equivalence relation in Z .
36. A relation R is defined in the set of integers Z by xRy iff $3x + 4y$ is a multiple of 7. Show that R is an equivalence relation in Z .
37. Consider the set $N \times N$, the set of ordered pairs of natural numbers. Let \sim be a relation in $N \times N$ defined by $(x, y) \sim (z, u)$ if $x + u = y + z$. Prove that \sim is an equivalence relation. Determine the equivalence class of $(1, 4)$.
38. Define congruence modulo n relation and prove that congruence modulo n is an equivalence relation in Z .
39. Show that the set of all 2×2 matrices with real numbers w.r.t. addition of matrices is a group.

Marks – 04 / 06

1. Let \sim be an equivalence relation on set A . Prove that any two equivalence classes are either disjoint or identical.
2. Prove that every equivalence relation on a non-empty set S induces a partition on S and conversely every partition of S defines an equivalence relation on S .
3. If $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$ for $a, b, c, d \in \mathbb{Z}$ and $n \in \mathbb{N}$ then prove that
 - i) $(a + c) \equiv (b + d) \pmod{n}$
 - ii) $(a - c) \equiv (b - d) \pmod{n}$
 - iii) $ac \equiv bd \pmod{n}$
4. Write the algorithm to find solution of linear congruence,
 $ax \equiv b \pmod{n}$, for $a, b \in \mathbb{Z}$ and $n \in \mathbb{N}$
5. State and prove Fermat's Theorem.
6. If G is a group then prove that
 - i) identity of G is unique
 - ii) Every element of G has unique inverse in G .
 - iii) $(a^{-1})^{-1} = a, \quad \forall a \in G$
7. If G is a group then prove that
 - i) identity of G is unique
 - ii) $(a^{-1})^{-1} = a, \quad \forall a \in G$
 - iii) $(ab)^{-1} = b^{-1}a^{-1}, \quad \forall a, b \in G$.
8. Let G be a group and $a, b, c \in G$. Prove that
 - i) $ab = ac \Rightarrow b = c$ left cancellation law.
 - ii) $ba = ca \Rightarrow b = c$ Right cancellation law.
9. Let G be a group and $a, b \in G$. Prove that the equations i) $ax = b$ ii) $ya = b$ have unique solutions in G
10. Let G be a group and $a \in G$. Prove that $(a^n)^{-1} = (a^{-1})^n, \quad \forall n \in \mathbb{N}$

11. Let G be a group and $a \in G$. For $m, n \in \mathbb{N}$,
 Prove that i) $a^m a^n = a^{m+n}$ ii) $(a^m)^n = a^{mn}$
12. Define an abelian group. If in a group G the order of every element (except identity element) is two then prove that G is an abelian group.
13. Solve the following linear congruence equations
 i) $3x \equiv 2 \pmod{8}$ ii) $6x \equiv 5 \pmod{9}$
14. Define a group. Show that any element $a \in G$ has a unique inverse in G .
 Further show that $(a*b)^{-1} = b^{-1}*a^{-1}$, $\forall a, b \in G$.
15. If R is an equivalence relation on a set A then for any $a, b \in A$, prove that
 i) $[a] = [b]$ or $[a] \cap [b] = \phi$
 ii) $\cup \{ [a] / a \in A \} = A$
16. If \sim is an equivalence relation on set A and $a, b \in A$ then show that
 i) $a \in [a]$ for all $a \in A$
 ii) $b \in [a]$ iff $[a] = [b]$
 iii) $a \sim b$ iff $[a] = [b]$
17. Define residue classes of integers modulo n . Show that the number of residue classes of integers modulo n are exactly n .

Unit – 04

Subgroups and Cyclic Groups

Marks – 02

- 1) Define subgroup. Give example
- 2) Define proper and improper subgroups. Give example.
- 3) Define a cyclic group. Give example.
- 4) Define left coset and right coset..
- 5) State Lagrange's Theorem.
- 6) State Fermat's Theorem.
- 7) State Euler's Theorem.
- 8) Show that $nZ = \{ nr / r \in Z \}$ is a subgroup of $(Z, +)$, where $n \in \mathbb{N}$.
- 9) Show that $(5Z, +)$ is a subgroup of $(Z, +)$
- 10) Is group (Q^+, \cdot) a subgroup of $(R, +)$? Justify.
- 11) Determine whether or not $H = \{ix : x \in R\}$ under addition is a subgroup of $G =$ group of complex numbers under addition.
- 12) Find all possible subgroups of $G = \{ 1, -1, i, -i \}$ under multiplication.
- 13) Find proper subgroups of $(Z, +)$
- 14) Write all subgroups of the multiplicative group of 6th roots of unity.
- 15) Find all proper subgroups of the group of non-zero reals under multiplication.
- 16) Give an example of a proper subgroup of a finite group.
- 17) Give an example of a proper finite subgroup of an infinite group.
- 18) Give an example of a proper infinite subgroup of an infinite group.
- 19) Is union of two subgroups a subgroup ? Justify.
- 20) Prove that cyclic group is abelian.
- 21) Show by an example that abelian group need not be cyclic.
- 22) Let $G = \{ 1, -1, i, -i \}$ be a group under multiplication and $H = \{ 1, -1 \}$ be its subgroup. Find all right cosets of H in G .
- 23) Find the order of each proper subgroup of a group of order 15. Are they cyclic.
- 24) Find generators of Z_6 under addition modulo 6.
- 25) Verify Euler's theorem by taking $m = 12, a = 7$.
- 26) Verify Lagrange's theorem for Z_9 under addition modulo 9.

- 27) Let $G = \mathbb{Z}$ be an additive group of integers and $H = 3\mathbb{Z}$ a subgroup of G then $H+2$ is
- A) $\{3, 6, 9, 12, \dots\}$ B) $\{2, 5, -1, 8, -4, \dots\}$
 C) $\{1, 2, 3, 4, 5, \dots\}$ D) $\{1, 4, -2, 7, -5, \dots\}$
- 28) Let $G = \{1, -1, i, -i\}$ be a group under multiplication and $H = \{1, -1\}$ is a subgroup of G then $H(-i)$ is
- A) $\{-1, 1\}$ B) $\{-i, i\}$ C) $\{i, -1\}$ D) $\{1, 1\}$
- 29) If $n = 12$ then $\phi(12)$ is
- A) 5 B) 4 C) 7 D) 12
- 30) If p is prime then generators of a cyclic group of order p ---
- A) p B) $p-1$ C) p^2 D) $p+1$
- 31) A cyclic group having only one generator can have at the most --- element
- A) 1 B) 3
 C) 2 D) None of these
- 32) In additive group \mathbb{Z}_{12} , $(\bar{4}) = \dots$
- A) $\{\bar{4}, \bar{8}\}$ B) $\{\bar{4}, \bar{8}, \bar{0}\}$ C) $\{\bar{2}, \bar{4}, \bar{0}\}$ D) \mathbb{Z}_{12}

Marks – 04

- 1) If H is a subgroup of G and $x \in G$, show that $xHx^{-1} = \{ xhx^{-1} / h \in H \}$ is a subgroup of G
- 2) Let G be a group. Show that $H = Z(G) = \{ x \in G / xa = ax, \forall a \in G \}$ is a subgroup of G .
- 3) Let G be an abelian group with identity e and $H = \{ x \in G / x^2 = e \}$. Show that H is a subgroup of G .
- 4) Let G be the group of all non-zero complex numbers under multiplication. Show that $H = \{ a+ib \in G / a^2+b^2 = 1 \}$ is a subgroup of G
- 5) Show that $H = \{ x \in G : xb^2 = b^2x, \forall b \in G \}$ is subgroup of G .
- 6) Write all subgroups of the multiplicative group of non-zero residue classes modulo 7.
- 7) Determine whether $H_1 = \{ \bar{0}, \bar{4}, \bar{8} \}$ and $H_2 = \{ \bar{0}, \bar{5}, \bar{10} \}$ are subgroups of $(Z_{12}, +_{12})$
- 8) Let G be a finite cyclic group of order n , and $G = \langle a \rangle$. Show that $G = \langle a^m \rangle \Leftrightarrow (m, n) = 1$, where $0 < m < n$
- 9) Find all subgroups of $(Z_{12}, +_{12})$.
- 10) Find all subgroups of (Z_7^1, X_7)
- 11) Find all generators of additive group Z_{20}
- 12) Let $G = \{ 1, -1, i, -i \}$ be a group under multiplication and $H = \{ 1, -1 \}$ be it's subgroup. Find all right coset of H in G
- 13) Compute the right cosets of $4Z$ in $(Z, +)$.
- 14) Let $Q = \{ 1, -1, i, -i, j, -j, k, -k \}$ be a group under multiplication and $H = \{ 1, -1, i, -i \}$ be its subgroup. Find all the right and left cosets of H in G
- 15) Let $G \cong (Z_8, +_8)$ and $H = \{ \bar{0}, \bar{4} \}$. Find all right cosets of H in G
- 16) Let H be a subgroup of a group G and $a \in G$. Show that $Ha = \{ x \in G / xa^{-1} \in H \}$
- 17) Let $G = \{ 1,2,3,4,5,6,7,8,9,10 \}$. Show that G is a cyclic group under multiplication modulo 11. Find all its generators, all its subgroups and order of every element. Also verify the Lagrange's theorem.

- 18) List all the subgroups of a cyclic group of order 12.
- 19) Find order of each element in $(Z_7, +_7)$
- 20) If Z_8 is a group w.r.t. addition modulo 8
 - i) Show that Z_8 is cyclic.
 - ii) Find all generators of Z_8
 - iii) Find all proper subgroups of Z_8
- 21) Show that every proper subgroup of a group of order 35 is cyclic.
- 22) Show that every proper subgroup of a group of order 77 is cyclic.
- 23) Let G be a group of order 17. Show that for any $a \in G$ either $o(a) = 1$ or $o(a) = 17$.
- 24) Let A, B be subgroups of a finite group G , whose orders are relatively prime. Show that $A \cap B = \{e\}$.
- 25) Find the order of each element in the group $G = \{1, w, w^2\}$, where w is complex cube root of unity, under usual multiplication.
- 26) Find all subgroups of group of order 41. How many of them are proper?
- 27) Find the remainder obtained when 3^{54} is divided by 11.
- 28) Find the remainder obtained when 33^{19} is divided by 7.
- 29) Using Fermat's theorem, find the remainder when
 - i) 9^{87} is divided by 13.
 - ii) $5^{41} + 41^{12}$ is divided by 13
- 30) Find the remainder obtained when 15^{27} is divided by 8.

Marks – 04 / 06

- 1) A non-empty subset H of a group G is a subgroup of G iff
 $a, b \in H \Rightarrow ab^{-1} \in H$.
- 2) A non-empty subset H of a group G is a subgroup of G iff
 - i) $a, b \in H \Rightarrow ab^{-1} \in H$
 - ii) $a \in H \Rightarrow a^{-1} \in H$.
- 3) Prove that Intersection of two subgroups of a group is a subgroup
- 4) Let H, K be subgroups of a group G . Prove that $H \cup K$ is a subgroup of G , iff either $H \subseteq K$ or $K \subseteq H$
- 5) Show that every cyclic group is abelian. Is the converse true? Justify.
- 6) Show that If G is a cyclic group generated by a , then a^{-1} also generated by G .
- 7) Show that every subgroup of a cyclic group is cyclic.
- 8) Let H be a subgroup of a group G . prove that
 - i) $a \in H \Leftrightarrow Ha = H$
 - ii) $a \in H \Leftrightarrow aH = H$
- 9) Let H be a subgroup of a group G . Prove that
$$Ha = Hb \Leftrightarrow ab^{-1} \in H$$
$$aH = bH \Leftrightarrow b^{-1}a \in H, \quad \forall a, b \in G.$$
- 10) Let H be a subgroup of a group G . Prove that
 - i) Any two right cosets of H are either disjoint or identical.
 - ii) Any two left cosets of H are either disjoint or identical.
- 11) If H is a subgroup of a finite group G . Then prove that $0(H) / 0(G)$
- 12) Prove that every group of prime order is cyclic and hence abelian
- 13) Order of every element 'a' of a finite group G is a divisor of order of a group i.e. $0(a) / 0(G)$
- 14) If a is an element of a finite group G then $a^{o(G)} = e$
- 15) If an integer a is relatively prime to a natural number n then prove that
$$a^{\phi(n)} \equiv 1 \pmod{n}, \quad \phi \text{ being the Euler's function.}$$
- 16) Prove that If p is a prime number and a is an integer such that $P \nmid a$, then $a^{p-1} \equiv 1 \pmod{p}$

Unit - 05

De-moiver's Theorem, Elementary Functions.

Marks – 02

- 1) State De-Moiver's Theorem for integral indices.
- 2) List n – n th roots of unity.
- 3) Write 3- distinct cube roots of unity.
- 4) Find the sum of all n - n th roots of unity.
- 5) Simplify $(\cos 3\theta + i \sin 3\theta)^8 \cdot (\cos 4\theta - i \sin 4\theta)^{-2}$
- 6) Simplify $\frac{(1+i)(1+\sqrt{3}i)}{i(1-\sqrt{3}i)}$, using De-Moiver's Theorem.
- 7) Find 4- fourth roots of unity.
- 8) Solve the equation $x^2 - i = 0$, using De-Moiver's Theorem.
- 9) Separate into real and imaginary parts of $e^{5 + \frac{\pi}{2}i}$
- 10) Separate into real and imaginary parts of $e^{(5+3i)^2}$
- 11) Define $\sin z$ and $\cos z$, $z \in \mathbb{C}$.
- 12) Define $\sinh z$ and $\cosh z$, $z \in \mathbb{C}$.
- 13) Prove that $\cos^2 z + \sin^2 z = 1$, using definitions of $\cos z$ and $\sin z$.
- 14) Prove that $\tan z = \frac{2 \tan z}{1 - \tan^2 z}$
- 15) Prove that $\sin iz = i \sinh z$
- 16) Prove that $\sinh (iz) = i \sin z$
- 17) Prove that $\cos (iz) = \cosh z$
- 18) Prove that $\cosh (iz) = \cos z$
- 19) Prove that $\tanh (iz) = i \tan z$
- 20) Prove that $\tan (iz) = i \tanh z$

21) The four fourth roots of unity are ----, ----, ---- and ----

22) If $z = \sqrt{3} - i$, then $z^{12} =$ ----

23) $e^{-\pi i} =$ ----, and $e^{4\pi i} =$ ----

24) Period of $\sin z$ is ----

Period of $\cos z$ is ----

25) Period of $\sinh z$ is ----

Period of $\cosh z$ is ----

26) Express $\frac{(\sqrt{3}-i)^2}{(1+i)^{10}}$ in the form $p + iq$ where p, q are reals.

27) $(\cos\theta + i\sin\theta)^7$ has seven distinct values.

T F

28) $(\cos\theta + i\sin\theta)^{3/4}$ has 4 distinct values.

T F

29) $\operatorname{Re}(e^z) = e^{\operatorname{Re}(z)}$

T F

30) $|e^z| = e^{|z|}$

T F

31) Match

a) $\sinh^2 z + \cosh^2 z$

i) 1

b) $\sinh^2 z - \cosh^2 z$

ii) -1

c) $i \sin(iz)$

iii) e^z

d) $\sec z \cdot \cos z$

iv) $-\sinh z$

v) $\frac{2}{e^{iz} - e^{-iz}}$

Choose the correct answer (32 to 40)

32) Consider

- a) The sum of the n , n th roots of unity is always 1
- b) The product of any two roots of unity is a root of unity.

- A) Both a) & b) are true
- B) Only a) is true
- C) Only b) is true
- D) Both are false

33) A value of $\log i$ is

- A) πi
- B) $\pi i / 2$
- C) 0
- D) $-\pi i / 2$

34) The real part of $\sin (x + iy)$ is

- A) $\sin x \cdot \cosh y$
- B) $\cos x \cdot \sinh y$
- C) $\sinh x \cdot \cos y$
- D) $\cosh x \cdot \sin y$

35) 2π is period of

- A) $\cos z$
- B) $\tan z$
- C) e^z
- D) $\cot z$

36) a) $\cos (iz) = \cosh z$ b) $\sin (iz) = i \sinh z$

- A) Both are true
- B) Both are false
- C) Only a) is true
- D) Only b) is true

37) $\sinh^2 z - \cosh^2 z$ is equal to

- A) $\cosh 2z$
- B) 1
- C) -1
- D) $\sinh 2z$

38) If w is an imaginary 9^{th} root of unity, then $w + w^2 + \dots + w^8$ is equal to

- A) 9
- B) 0
- C) 1
- D) -1

39) A square root of $2i$ is

- A) $1 - i$
- B) $1 + i$
- C) $\sqrt{2}$
- D) $\sqrt{2} i$

40) $(\cos \pi/4 + i \sin \pi/4)^{-2}$ is

- A) i
- B) $-i$
- C) 1
- D) -1

Marks - 04

1. Simplify using De-Moivre's Theorem, the expression

$$\frac{(\cos 2\theta - i \sin 2\theta)^7 (\cos 3\theta + i \sin 3\theta)^{-5}}{(\cos 4\theta + i \sin 4\theta)^{12} (\cos 5\theta - i \sin 5\theta)^{-6}}$$

2. Simplify

$$\frac{(\cos \theta + i \sin \theta)^{8/7} (\cos \theta - i \sin \theta)^{12/7}}{(\cos \theta + i \sin \theta)^{12/7} (\cos 4\theta - i \sin 4\theta)^{5/4}}$$

3. Prove that $\left[\frac{1 + \sin \theta + i \cos \theta}{1 + \sin \theta - i \cos \theta} \right]^n = \cos [(\pi/2 - \theta)n] + i \sin [(\pi/2 - \theta)n]$

4. If α and β are roots of $x^2 - 2x + 2 = 0$ and n is a positive integer, then prove that

$$\alpha^n + \beta^n = 2^{\frac{n+2}{2}} \cos(n\pi/4)$$

5. Evaluate $(1 + i\sqrt{3})^{10} + (1 - i\sqrt{3})^{10}$

6. Prove that $(1 + i\sqrt{3})^{-10} = 2^{-11}(-1 + i\sqrt{3})$

7. Prove that $(-1 + i)^7 = -8(1 + i)$

8. Prove that $(1 + i\sqrt{3})^8 + (1 - i\sqrt{3})^8 = -256$

9. If $x = \cos \alpha + i \sin \alpha$, $y = \cos \beta + i \sin \beta$ prove that

$$\frac{x - y}{x + y} = i \tan \left(\frac{\alpha - \beta}{2} \right)$$

10. Find $(3 + 4i)^{1/2} + (3 - 4i)^{1/2}$

11. Find all values of $(1 - i\sqrt{3})^{1/4}$

12. Find all values of $(1 + i)^{1/5}$

Show that their continued product is $1 + i$.

13. Find the continued product of the four values of $\left(\frac{1}{2} + i \frac{\sqrt{3}}{2} \right)^{3/4}$

14. If w is a complex cube root of unity, prove that $(1 - w)^6 = -27$

Using De-Moiver's Theorem, solve the following equations (15 to 25)

15. $x^4 - x^3 + x^2 - x + 1 = 0$

16. $x^4 + x^3 + x^2 + x + 1 = 0$

17. $x^8 - x^4 + 1 = 0$

18. $x^9 - x^5 + x^4 - 1 = 0$

19. $x^{10} + 11x^5 + 10 = 0$

20. $16x^4 - 8x^3 + 4x^2 - 2x + 1 = 0$

21. $x^3 + x^2 + x + 1 = 0$

22. $x^6 - 1 = 0$

23. $x^4 + 1 = 0$

24. $z^7 - z^4 + z^3 - 1 = 0$

25. $z^{12} - z^6 + 1 = 0$

26. Express $\cos^5\theta$ in terms of cosines of multiple of angle θ .

27. Express $\cos^6\theta$ in terms of cosines of multiple of angle θ .

28. Express $\sin^5\theta$ in terms of sines of multiple of angle θ .

29. Prove that $\cos^8\theta = 1/128 [\cos 8\theta + 8 \cos 6\theta + 28 \cos 4\theta + 56 \cos 2\theta + 35]$

30. Prove that $\cos^7\theta = 1/64 [\cos 7\theta + 7 \cos 5\theta + 21 \cos 3\theta + 35 \cos \theta]$

31. Prove that $\sin 7\theta = 7\cos^6\theta \sin\theta - 35 \cos^4\theta \sin^3\theta + 21 \cos^2\theta \sin^5\theta - \sin^7\theta$

32. Prove that $\cos 5\theta = \cos^5\theta - 10 \cos^3\theta \sin^2\theta + 5 \cos\theta \sin^4\theta$

33. Prove that $\sin 5\theta = 5 \cos^4\theta \sin\theta - 10 \cos^2\theta \sin^3\theta + \sin^5\theta$

34. If $\sin \alpha + \sin \beta + \sin \gamma = \cos \alpha + \cos \beta + \cos \gamma$ prove that

a) $\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3 \sin (\alpha + \beta + \gamma)$

b) $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3 \cos (\alpha + \beta + \gamma)$

35. Express $\frac{\sin 7\theta}{\sin \theta}$ in powers of $\sin \theta$ only.

36. Prove that $\frac{\sin 6\theta}{\sin \theta} = 32 \cos^5\theta - 24\cos^3\theta + 6 \cos\theta$

37. Prove that $\frac{\sin 6\theta}{\cos \theta} = 32 \sin^5\theta - 32 \sin^3\theta + 6 \sin\theta$

38. Using definitions of $\cos z$ and $\sin z$, prove that $\sin^2 z + \cos^2 z = 1$
39. If z_1 and z_2 are complex numbers, show that

$$\cos (z_1 + z_2) = \cos z_1 \cos z_2 - \sin z_1 \sin z_2$$
40. Prove that $\cosh (z_1 + z_2) = \cosh z_1 \cosh z_2 + \sinh z_1 \sinh z_2$
41. Prove that a) $2\cosh^2 z - 1 = \cosh 2z$
 b) $2\sinh^2 z + 1 = \cosh 2z$
42. Find the general values of a) $\text{Log}(-i)$ b) $\text{Log}(-5)$

Separate into real and imaginary parts of (43 to 55)

43. $\log (4 + 3i)$
44. $\log (3 + 4i)$
45. $\sin (x + iy)$
46. $\cos (x + iy)$
47. $\tan (x + iy)$
48. $\sec (x + iy)$
49. $\text{cosec} (x + iy)$
50. $\cosh (x + iy)$
51. $\text{coth} (x + iy)$
52. $\text{sech} (x + iy)$
53. $\text{cosech} (x + iy)$
54. $\tanh (x + iy)$
55. $\cot (x + iy)$

Prove the following (56 to 60)

56. $\sinh 2z = 2 \sinh z \cosh z$
57. $\sinh 2z = \frac{2 \tanh z}{1 - \tanh^2 z}$
58. $\cosh 2z = \frac{1 + \tanh^2 z}{1 - \tanh^2 z}$
59. $\tanh 2z = \frac{2 \tanh z}{1 + \tanh^2 z}$

60. $\cosh 3z = 4 \cosh^2 z - 3 \cosh z$
61. If $\cos (x + iy) = \cos \alpha + i \sin \alpha$ show that $\cos 2x + i \cosh 2y = 2$
62. If $\sin (x + iy) = \tan \alpha + i \sec \alpha$ show that $\cos 2x \cosh 2y = 3$
63. If $\sin (\alpha + i\beta) = x + iy$, prove that
- $$\frac{x^2}{\cosh^2 \beta} + \frac{y^2}{\sinh^2 \beta} = 1 \text{ and } \frac{x^2}{\sin^2 \alpha} - \frac{y^2}{\cos^2 \alpha} = 1$$
64. If $x + iy = \cosh (u + iv)$, show that
- $$\frac{x^2}{\cosh^2 u} + \frac{y^2}{\sinh^2 u} = 1 \text{ and } \frac{x^2}{\cos^2 v} - \frac{y^2}{\sin^2 v} = 1$$
65. If $x + iy = \cosh (u + iv)$, show that $x^2 \operatorname{sech}^2 u + y^2 \operatorname{cosech}^2 u = 1$
66. If $x + iy = \cosh (u + iv)$, show that $(1 + x)^2 + y^2 = (\cosh v + \cos u)^2$
67. If $x + iy = \cos (u + iv)$, show that $(1 - x)^2 + y^2 = (\cosh v - \cos u)^2$
68. If $\cos (x + iy) = r(\cos \alpha + i \sin \alpha)$, show that $2y = \log \left[\frac{\sin (x - \alpha)}{\sin (x + \alpha)} \right]$
69. If $u = \log \left[\tan \left(\frac{\pi}{4} + \frac{x}{2} \right) \right]$, prove that $\tanh \frac{u}{2} = \tan \frac{x}{2}$
70. If $\tan (x + iy) = A + iB$ then show that $\frac{A}{B} = \left[\frac{\sin 2x}{\sinh 2y} \right]$
71. Prove that $\sin[\log (i^i)] = -1$
72. Show that $\sin \left[i \log \left(\frac{1 + ie^{-i\theta}}{1 - ie^{-i\theta}} \right) \right]$ is purely real.
73. Find the 5-5th roots of -1.
74. Find the modulus and principal value of the argument of $\frac{(1 + i\sqrt{3})^7}{(\sqrt{3} - i)^{11}}$
75. Express $\frac{(\sqrt{3} - i)^7}{(1 + i)^{10}}$ in the form $a + ib$, where a and b are reals.
76. If $z = -(\sqrt{3} + i)$, find z^{10}
77. If $x_i^2 + 1 = 2 x_i \cos \theta$ ($i = 1, 2, 3$), then prove that one of the value of $x_1 x_2 x_3$
 $+ \frac{1}{x_1 x_2 x_3}$ is $2 \cos (\theta_1 + \theta_2 + \theta_3)$

78. If $2 \cos \alpha = x + \frac{1}{x}$ and $2 \cos \beta = y + \frac{1}{y}$, prove that one of the values of

$$x^m y^n + \frac{1}{x^m y^n} \text{ is } 2 \cos (m\alpha + n\beta)$$

79. If $2 \cos \theta = x + \frac{1}{x}$ and $2 \cos \phi = y + \frac{1}{y}$, prove that

$$\frac{x^m}{y^n} - \frac{y^n}{x^m} = 2i \sin (m\theta - n\phi)$$

80. Solve the equation $x^2 - i = 0$, using De-moivre's theorem.

Marks - 04 / 06

- 1) State and prove De-Moiver's Theorem for integral indices.
- 2) State and prove De-Moiver's Theorem for rational indices.
- 3) State De-Moiver's Theorem. Obtain the formula for n-nth roots of unity.
- 4) Find n-nth roots of unity and represent them geometrically.
- 5) Show that the product of any two roots of unity is the root of unity.
- 6) Show that the 7th roots of unity form a series in G.P. and find their sum.
- 7) Show that the sum of n-nth roots of unity is zero.
- 8) Find n-nth roots of a complex number $z = x + iy$.
- 9) Prove that
$$(x + iy)^{m/n} + (x - iy)^{m/n} = 2(x^2 + y^2)^{m/2n} \cdot \cos \left[\frac{m}{n} \tan^{-1} \left(\frac{y}{x} \right) \right]$$
- 10) If $2 \cos \theta = x + \frac{1}{x}$ and $2 \cos \phi = y + \frac{1}{y}$ then show that
$$\frac{x^m}{y^n} + \frac{y^n}{x^m} = 2 \cos (m\theta - n\phi)$$
- 11) Define $\sin z$, $\cos z$ and $\sinh z$, $\cosh z$. Prove that $\sin z$ and $\cos z$ are periodic functions with period 2π .
- 12) Define $\tan z$. Prove that $\tan z$ is a periodic function with period π .
- 13) Define $\sinh z$, and $\cosh z$. Prove that $\sinh z$ and $\cosh z$ are periodic functions with period $2\pi i$.
- 14) Obtain the relation between circular functions $\sin z$, $\cos z$ and hyperbolic functions $\sinh z$, $\cosh z$.
- 15) Define $\text{Log } z$, $z \in \mathbb{C}$ Separate $\text{Log } z$ into real and imaginary parts.
- 16) Prove that $i \log \left[\frac{x-i}{x+i} \right] = \pi - 2 \tan^{-1} x$
- 17) Prove that $\cos \left\{ i \log \left(\frac{a+ib}{a-ib} \right) \right\} = \frac{a^2 - b^2}{a^2 + b^2}$
- 18) Prove that $\tan \left\{ i \log \left(\frac{a-ib}{a+ib} \right) \right\} = \frac{2ab}{a^2 - b^2}$
- 19) Using definition prove that $\cosh^2 z - \sinh^2 z = 1$
- 20) If $\sin^{-1}(\alpha + i\beta) = u + iv$, prove that $\sin^2 u$ and $\cosh^2 v$ are the roots of the quadratic equation $\lambda^2 - (1 + \alpha^2 + \beta^2) \lambda + \alpha^2 = 0$